Economics 451 Exam 1 Spring 2014 Prof Montgomery

Answer all questions. 100 points possible. Explanations can be brief.

1) [30 points] Following Becker's analysis of the household division of labor, consider a household in which a husband and wife can produce market goods (M) and household goods (H).

a) Suppose that the wife's production is given by the equations $M = 5\ t_M$ and $H = 5\ t_H$ where t_M = time spent in market production, t_H = time spent in household production, and the wife's time constraint is $t_M + t_H = 40$. Derive the equation for the wife's production possibilities curve.

b) Suppose the husband's production is given by the equations $M = 2\ t_M$ and $H = 4\ t_H$ where t_M = time spent in market production, t_H = time spent in household production, and the husband's time constraint is $t_M + t_H = 40$. Derive the equation for the husband's production possibilities curve.

c) Plot the overall household production possibilities curve. [HINT: Your graph doesn't need to be drawn perfectly to scale, but you must label your graph carefully and give numerical coordinates of important points on the curve.]

d) Suppose that the household's utility function has the functional form

$$U(M, H) = M + \theta H \quad \text{where } \theta \text{ is a constant such that } ½ < \theta < 1.$$

What level of market and household goods (M*, H*) will the household optimally choose? [HINT: What is the slope of any indifference curve?] What is the division of labor in the household? Does either household member split their time between sectors? How would your answers change if θ was lower so that $0 < \theta < ½$? Briefly explain.

2) [20 points] Consider a household with bilateral altruism between the husband and wife. Suppose that the utility functions for the husband (h) and wife (w) are

$$U_h = \ln(Z_h) + (1/3) \ln(Z_w) \quad \text{and} \quad U_w = \ln(Z_w) + (3/2) \ln(Z_h)$$

where ln( ) denotes natural log. Suppose that the husband has income I_h while the wife has income I_w. Each household member can transfer some of their income to the other member, but cannot enforce negative transfers. For each of the following cases, give the final consumption level of the husband and wife after any transfers have been made. [HINT: Total household income $I_h + I_w = 300$ in every case below.]

a) $I_h = 300, I_w = 0$ b) $I_h = 0, I_w = 300$ c) $I_h = 200, I_w = 100$ d) $I_h = I_w = 150$

3) [24 points] Consider a marriage market with 3 males and 3 females. Assume that utility is NOT transferable. Utilities generated by each possible match are given by the following matrix. The first column reports the payoff that each man would receive if he remains single; the first row reports the payoff that each female would receive if she remains single; the remaining entries reports the payoff (m_{ij}, f_{ji}) that would be received by male i and female j if they marry.

$$
\begin{array}{cc}
 & \text{females} \\
 & \begin{array}{cccc} \quad & 1 & 2 & 3 \end{array} \\
\text{males} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} . & 3 & 2 & 1 \\ 2 & (7,4) & (5,1) & (3,8) \\ 1 & (4,5) & (3,4) & (6,2) \\ 3 & (2,6) & (7,3) & (5,5) \end{bmatrix}
\end{array}
$$

a) Find an equilibrium match structure. Is there more than one equilibrium match structure? If so, report a second match structure. If not, briefly explain how you know there is only one equilibrium match structure.

b) Suppose female 3 receives a single payoff f_{30} equal to 4 (instead of 1). (All other utilities in the payoff matrix are unchanged.) Repeat the analysis of part (a): Find an equilibrium match structure. Then report a second equilibrium match structure, or else explain how you know there is only one equilibrium match structure.

4) [20 points] Now consider a marriage market with transferable utility. Suppose the surplus between matched couples depends solely on the ability of the male (A_m) and the ability of the female (A_f). In particular, suppose that the surplus s is given by

$$s = (A_m)^2 + (A_f)^2 - A_m A_f.$$

a) Given this surplus function, will assortative mating be positive or negative? Be sure to report the mathematical computation that determines your answer.

b) Given the surplus function above, suppose there are 2 males and 2 females. One male has low ability ($A_m = 1$) and one has high ability ($A_m = 2$). Similarly, one female has low ability ($A_f = 1$) and one has high ability ($A_f = 2$). What is the equilibrium match structure? Will it be possible to support the equilibrium match structure with an equal sharing rule (i.e., an equal split of the surplus within each match)? Briefly explain.

5) [6 points] In his chapter on altruism, Becker states that families with an altruistic member will have conflict over *distribution* but not *production*. Briefly explain.

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1a) [5 pts]

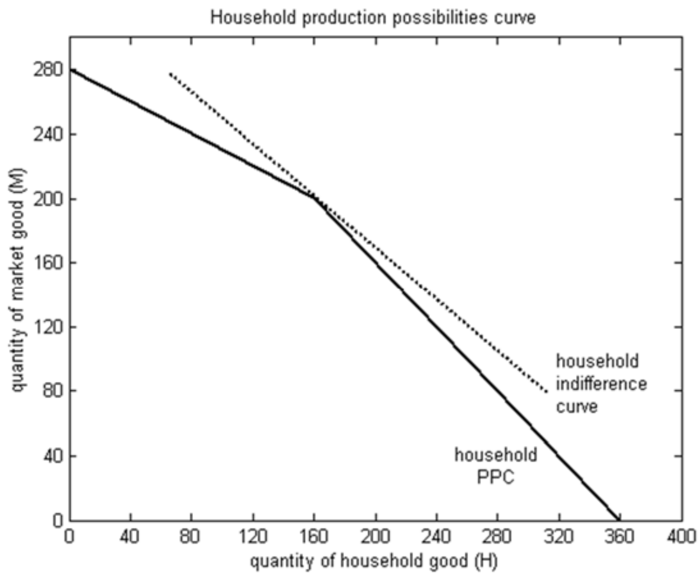
$t_M + t_H = 40 \rightarrow (M/5) + (H/5) = 40 \rightarrow M = 200 - H$

Note that the PPC should give M as a function of H (or vice versa).

b) [5 pts]

$t_M + t_H = 40 \rightarrow (M/2) + (H/4) = 40 \rightarrow M = 80 - H/2$

c) [8 pts]



d) [12 pts] The equation for an indifference curve is $M = U - \theta H$ (where U is the fixed utility level corresponding to the indifference curve). Thus, any indifference curve is linear with slope $-\theta$ (between $-1/2$ and -1). Consequently, the household maximizes utility by choosing M* = 200 and H* = 160 as shown on the graph in part (c). Thus, the wife specializes completely in market production and the husband specializes completely in household production.

If θ was lower (between 0 and ½), the household's indifference curves would be flatter than either segment of the PPC; the household would maximize utility by choosing M* = 280 and H* = 0; both the wife and husband would specialize completely in market production.

2a) [6 pts] If the husband controlled all household income, he would choose Z_h to solve

$$\text{maximize} \quad \ln(Z_h) + (1/3)\ln(300 - Z_h)$$

which implies $(1/Z_h) + (1/3)(1/(300-Z_h))(-1) = 0$

hence $Z_h^* = 225$ and $Z_w^* = 75$.

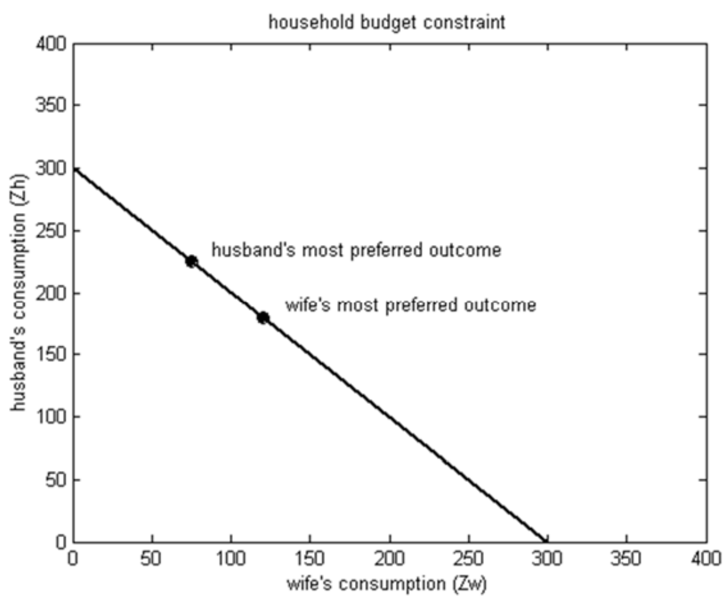
b) [6 pts] If the wife controlled all household income, she would choose Z_w to solve

$$\text{maximize} \quad \ln(Z_w) + (3/2)\ln(300 - Z_w)$$

which implies $(1/Z_w) + (3/2)(1/(300-Z_w))(-1) = 0$

hence $Z_w^* = 120$ and $Z_h^* = 180$.

Graphically, we could plot the solutions from parts (a) and (b):



c) [4 pts] Neither spouse's altruism is effective, so there would be no transfers: $Z_h = 200$ and $Z_w = 100$. (Note that this endowment point is between the husband's and wife's most preferred outcomes on the graph above.)

d) [4 pts] The wife's altruism is effective while the husband's is not. We thus obtain the outcome from part (b): $Z_h = 180$ and $Z_w = 120$. (Note that the endowment $Z_h = Z_w = 150$ is to the "southeast" of the wife's most preferred outcome on the graph above. Thus, the wife increases her own utility by transferring 30 to the husband.)

3a) [12 pts] Using the Gale-Shapley algorithm, we obtain the equilibrium match structure {M1-F1, M2-F3, M3-F2} when males make the offers, and obtain another equilibrium match structure {M1-F3, M2-F1, M3-F2} when females make the offers.

b) [12 pts] With the change in female 3's single payoff, the Gale-Shapley algorithm generates the match structure {M1-F3, M2-F1, M3-F2} when either the males or the females make the offers. Gale and Shapley proved that the best equilibrium match structure for males is the structure that arises when males make the offers. Conversely, the worst equilibrium match structure for males is the structure that arises when females make the offers. In the present case, because the best and worst equilibrium match structures are the same, there can be no more than one equilibrium match structure.

4a) [8 pts] Assortative mating is positive (i.e., matches are between individuals with similar ability levels) when the cross-partial derivative $\partial^2 s/\partial A_m \partial A_f$ is positive, and assortative mating is negative (i.e., high ability individuals marry lower ability individuals) when this condition is negative. Here,

$$\partial^2 s/\partial A_m \partial A_f \;=\; \partial[\partial s/\partial A_m]/\partial A_f \;=\; \partial[2A_m - A_f]/\partial A_f \;=\; -1$$

hence assortative mating is negative.

b) [12 pts] Assuming that male 1 and female 1 are low ability (A = 1) and male 2 and female 2 are high ability (A = 2), the surplus matrix is given by

$$\begin{array}{ccc} & & \text{females} \\ & & \begin{array}{cc} 1 & 2 \end{array} \\ \text{males} & \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \end{array}$$

There are two possible match structures: {M1-F1, M2-F2} generates aggregate surplus 5, while {M1-F2, M2-F1} generates aggregate surplus 6. Thus, {M1-F2, M2-F1} is the equilibrium match structure because it maximizes aggregate surplus. To support this match structure, the shares received by the males (θ_{12}^*, θ_{21}^*) must satisfy the inequalities

$$1 \;<\; \theta_{12}^*\, 3 + (1-\theta_{21}^*)\, 3 \qquad \text{and} \qquad 4 \;<\; \theta_{21}^*\, 3 + (1-\theta_{12}^*)\, 3$$

Equal shares ($\theta_{12}^* = \theta_{21}^* = ½$) would violate the second inequality (implying that male 2 and female 2 would leave their current partners for each other). Shares need to be biased toward the high-ability spouses ($1/3 < \theta_{21}^* - \theta_{12}^* < 2/3$) to support the equilibrium.

5) [6 pts] A selfish individual would always prefer a larger transfer from the altruist. Given that selfish members always want more for themselves, there is conflict over "distribution" in the household. However, as described by the Rotten Kid Theorem, selfish individuals will not take actions that would decrease household income (even if those actions would raise their own income). In this sense, there is no conflict over "production" in the household. [See Becker, p 292, for more discussion.]

Economics 451 Exam 2 Spring 2014 Prof Montgomery

Answer all 4 questions. 130 points possible. Explanations can be brief.

1. [24 points] Five voters have preferences over 4 outcomes (A, B, C, D):

voter 1's preferences are A > B > C > D (where > denotes strict preference)
voter 2's preferences are B > A > D > C
voter 3's preferences are C > D > A > B
voter 4's preferences are D > B > C > A
voter 5's preferences are C > B > A > D

a) Determine the collective preference order using the Borda count procedure, showing the relevant computations.

b) Suppose that voter 2's preferences become B > A > C > D and voter 5's preferences become C > D > B > A while the preferences of the other three voters are unchanged from part (a). Again determine the collective preference order using the Borda count procedure, showing the relevant computations.

c) Briefly state condition I (independence from irrelevant alternatives) from Arrow's Impossibility Theorem. Does the Borda count procedure satisfy this condition? Explain, using your answers from parts (a) and (b) to illustrate.

2. [38 points] For the two cases below, the Condorcet procedure yielded the following collective preferences over the outcomes A, B, C, D, and E:

(case 1) A > B, A > C, A > D, A > E, B > C, B > E, C > D, C > E, D > B, D > E

(case 2) A > B, A > C, A > E, B > C, B > D, C > D, C > E, D > A, D > E, E > B

Answer each of the following questions for case 1 and then answer again for case 2:

a) Are the collective preferences transitive? If so, state the collective ranking. If not, explain why transitivity doesn't hold. Which outcomes are in the Condorcet set ("top cycle")? Show that each outcome in this set satisfies the relevant criterion for being in the top cycle.

b) Suppose everyone votes sincerely using amendment procedure (so that two outcomes are compared, the winner is then compared to a third outcome; the winner is then compared to a fourth outcome, etc). Is it possible to structure an agenda so that B is the winner? If so, show the agenda (using a "final four" type diagram). Otherwise, explain why this is not possible.

c) Suppose everyone votes strategically using successive procedure (so that each outcome is voted up or down, and voting ceases as soon as one outcome is voted up). Is it possible to structure an agenda so that B is the winner? If so, show the agenda (using a game tree). Otherwise, explain why this is not possible.

3. [50 points] Consider an election with 4 candidates (A, B, C, D) and 9 million voters:

> 2 million Democrats with preferences A > B > C > D
> 1 million Democrats with preferences B > A > C > D
> 1 million independents with preferences B > C > A > D
> 2 million Republicans with preferences C > B > D > A
> 3 million Republicans with preferences D > C > B > A

a) Use the Condorcet procedure to derive collective preferences.

b) Report the winset W(i) for each candidate $i \in \{A, B, C, D\}$

c) Explain the "single-peaked preference" condition. Do all voters have single-peaked preferences in this problem? What implications does this have for the outcome of the Condorcet procedure and the winsets? Use your answers from parts (a) and (b) to illustrate.

Instead of a general election that includes all four candidates, suppose that each party holds a primary, and the winners of the primaries then compete in the general election. More precisely, suppose that Democratic voters choose between the two Democratic candidates (A and B). Then, Republican voters choose between the two Republican candidates (C and D). Then, the general election is held. Assume that independent voters do not vote in either primary, but do vote in the general election.

d) Assume sincere ("naïve") voting in the primaries. Which candidate would win each primary? Which candidate would win the general election?

e) Assume strategic ("sophisticated") voting in the primaries. Which candidate would win each primary? Which candidate would win the general election? Briefly explain, using a game tree diagram to describe and analyze this problem.

4) [18 points] Stigler and Becker (*American Economic Review* 1977) proposed an economic model of drug addiction. Is that model consistent with the assumption of stable preferences? Briefly explain. Assuming that individuals are myopic (ignoring the implications of their present actions for their future "euphoric capital"), what condition on the individual's preferences determines whether the individual becomes an addict (i.e., continues to use drugs even though euphoric capital is decreasing over time)? Briefly explain using an indifference-curve graph.

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1a) [6 pts] The Borda count procedure uses each voter's preferences to assign points to each outcome, and then derives the collective preference order by summing these points across voters. More precisely, given n outcomes, a voter's top choice receives n-1 points, her second choice receives n-2 points, etc. For this problem, adding points across voters, the point totals are

A receives 3 + 2 + 1 + 0 + 1 = 7 points
B receives 2 + 3 + 0 + 2 + 2 = 9 points
C receives 1 + 0 + 3 + 1 + 3 = 8 points
D receives 0 + 1 + 2 + 3 + 0 = 6 points

Thus, the collective preference order is B > C > A > D. (More generally, the Borda procedure can generate ties – two outcomes might receive the same number of total points – even if individual preferences are strict.)

b) [6 pts] The point totals are now 6 for A, 8 for B, 9 for C, 7 for D. Thus, the collective preferences are C > B > D > A.

c) [12 pts] Condition I requires that, if voters' preferences between outcomes i and j are fixed, then a change in their ranking of some outcome k (the "irrelevant alternative") should not affect the collective preference between i and j. The Borda count procedure does not satisfy condition I. For this problem, outcome D is the irrelevant alternative. Voter 2 moved D down his preference ranking; voter 5 moved D up his ranking; neither voters altered the ranking of the other outcomes. Thus, condition I would require the collective ranking B > C > A from part (a) to continue to hold in part (b). Instead, the Borda count procedure causes a reversal of collective preferences so that now C > B.

2a, case 1) [8 pts] Not transitive. There is a cycle B > C > D > B. The outcome A is the only outcome in the top cycle. A can defeat all other outcomes through the chain A > B > C > D > E (and other chains).

2a, case 2) [10 pts] Not transitive. All outcomes are included in the cycle A > E > B > C > D > A. This implies that all outcomes are in the top cycle. Given this cycle, A can defeat all other outcomes through the chain A > E > B > C > D, B can defeat all others through B > C > D > A > E, etc.

2b, case 1) [4 pts] No, there is no such agenda. Under amendment procedure with sincere voting, an outcome can win if and only if it is in the top cycle. B is not in the top cycle.

2b, case 2) [6 pts] Yes. One such agenda is: E vs A; winner vs D; winner vs C; winner vs B.

2c, case 1) [4 pts] No, there is no such agenda. Under successive procedure with sincere voting, an outcome can win if and only if it is in the top cycle. B is not in the top cycle.

2c, case 2) [6 pts] Yes. One such agenda is: vote up or down on B, then C, then D, then A (with E as the default outcome). Given sophisticated voting, the majority would vote up on B in this first round.

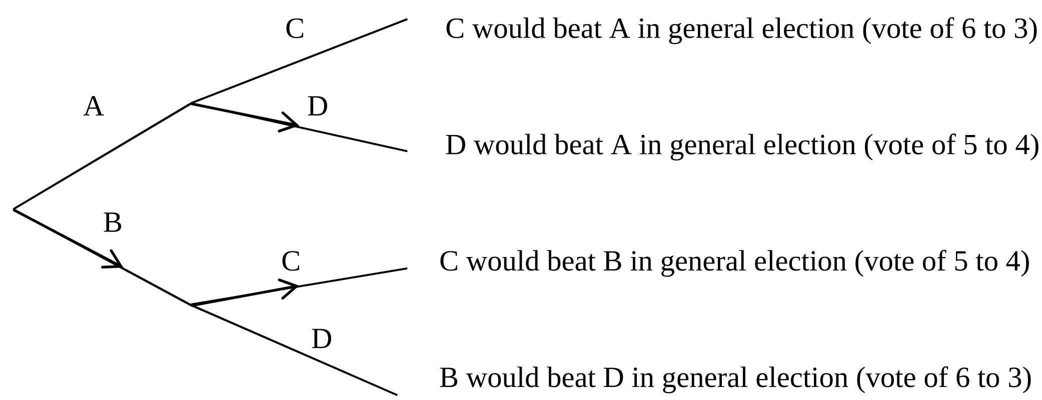
3a) [6 pts] The Condorcet procedure yields B > A, C > A, D > A, C > B, B > D, C > D.

b) [4 pts] W(A) = {B, C, D}, W(B) = {C}, W(C) = ∅, W(D) = {B, C}.

c) [18 pts] The single-peaked preference condition holds if all outcomes can be arranged on a one-dimensional continuum so that each voter's utility falls as the outcome moves further from his/her ideal point (most preferred outcome). Yes, preferences are single peaked in this problem (given outcomes arranged in the sequence A, B, C, D). Black's Single-Peakedness Theorem thus implies that collective preferences are transitive. Accordingly, the collective preferences in part (a) imply C > B > D > A. Black's Median Voter Theorem thus implies that the median voter's ideal point has an empty winset. The median voter in this problem has the ideal point C, and part (b) confirms that W(C) = ∅. [To determine the position of the median voter, imagine lining up all 9 million voters in the order of their ideal points (A then B then C then D). The median voter (number 4,500,000 in line) has ideal point C.]

d) [6 pts] Candidate A would win the Democratic primary (by a vote of 2 to 1). Candidate D would win the Republican primary (by a vote of 3 to 2). Candidate D would win the general election (by a vote of 5 to 4).

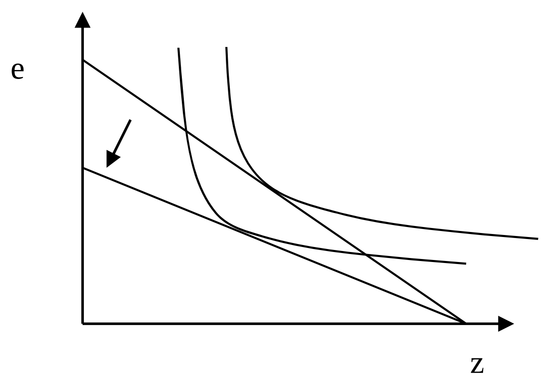
e) [16 pts] Consider the game tree below. Following the timing given in the problem, the first branches indicate the winner of the Democratic primary, the second branches indicate the winner of the Republican primary, and the terminal nodes indicate the winner of the general election.



Suppose voters are sophisticated. If A wins the Democratic primary, Republicans will vote for D in the Republican primary (who will go on to win the general election). If B wins the Democratic primary, Republicans will vote for C in the Republican primary (who will go on to win the general election). [While Republicans prefer D to C, they realize that D would lose the general election to B.] Anticipating the potential outcomes of the Republican primary, the Democrats will vote for B. [While Democrats prefer A to B, they prefer the general election is won by C rather than D.]

4) [18 pts] Stigler and Becker assume that individuals have a stable utility function U(e,z) where e represents the individual's consumption of "euphoria" and z represents his consumption of other goods. In turn, euphoria is produced from "euphoric capital" (S) and drugs (d) according to the equation $e = S \times d$. Thus, the production function for euphoria is changing over time, but preferences are stable.

An individual becomes an addict if e and z are poor substitutes, so that his indifference curves are sharply curved ("L-shaped") as shown below. In this case, the individual will decrease his consumption of other goods (and hence increase his consumption of drugs) even as his stock of euphoric capital decreases. [Using the version of the Stigler-Becker model from lecture, I'm assuming the individual faces an income constraint $d + z = I$.] Graphically,



Note that the constraint rotates downward as euphoric capital falls, so that a decrease in euphoric capital resembles (on the indifferences curve diagram) an increase in the "price" of euphoria. Because e and z are poor substitutes, the income effect (which causes the individual's consumption of z to fall) dominates the substitution effect (which causes his consumption of z to rise). If e and z were better substitutes (so that the indifference curves were more linear) then the substitution effect would dominate the income effect. Thus, the individual's consumption of z would rise (and hence his consumption of drugs would fall) as the stock of euphoric capital decreases.

Economics 451 Exam 3 Spring 2014 Prof Montgomery

Answer all 4 questions. 140 points possible. Explanations should be brief.

1) [50 points] Consider a factory where each worker chooses effort and also an identity as a "slacker" (S) or "hard worker" (H). The utility associated with each identity is

$$U(S) = p\,[10\,w\,e - e^2] + (1-p)\,[I(S) - (e - e(S))^2]$$

$$U(H) = p\,[10\,w\,e - e^2] + (1-p)\,[I(H) - (e - e(H))^2]$$

where p is the weight placed on non-identity concerns
w is the worker's wage per unit effort
e is the worker's effort choice
I(S) is the value of the S identity
I(H) is the value of the H identity
e(S) is the ideal effort for the S identity
e(H) is the ideal effort for the H identity

a) Suppose a worker places no weight on identity concerns so that p = 1. What is the optimal effort chosen by the worker (as a function of the wage w)?

b) Now suppose p = 0.4, I(S) = 5, e(S) = 5, I(H) = 0, and e(H) = 15. What is the optimal effort choice by slackers (as a function of the wage w) and the optimal effort choice made by hard workers (as a function of the wage w)?

c) Using your answers to parts (b), consider a worker with wage w = 2. What is the maximum utility possible for each identity? What identity and effort level is chosen by the worker?

d) Using your answers to part (b), consider a worker with wage w = 3. What is the maximum utility possible for each identity? What identity and effort level is chosen by the worker?

e) Suppose factory management creates an employee-of-the-month program to try to increase workers' effort choices. This program raises I(H) from 0 to 3 (while all other parameters remain fixed at the levels given in part b). Again derive the optimal effort and identity for workers with wage w = 2 and workers with wage w = 3, reporting all relevant computations. Did this program have the intended effect? Briefly discuss.

2) [40 points] The manager for a rock band needs to choose a venue for the band's next concert, but is uncertain about the demand for tickets. A smaller venue would be better if demand is lower, but a larger venue would be better if demand is higher. The following table reports the band's profit for three possible venues (small club, medium theater, large stadium) given three possible levels of demand (low, moderate, high).

	low demand	moderate demand	high demand
small club	15	15	15
medium theater	–15	60	60
large stadium	–175	–100	200

a) Suppose the manager believes there's a 1/5 chance that ticket demand is low, a 2/3 chance that ticket demand is moderate, and 2/15 chance that ticket demand is high. What is the expected profit for each venue? Which venue is chosen?

b) Now suppose the manager faces complete ambiguity and hence cannot assess probabilities for ticket demand. What is the value of each venue using the max-max criterion? Which venue is chosen? What is the value of each venue using the max-min criterion? Which venue is chosen?

c) What is the value of each venue using the Hurwicz criterion? What additional parameter is incorporated in the Hurwicz criterion? Determine the (numerical) range of this parameter for which each venue is chosen.

d) Briefly discuss the approach toward partial ambiguity taken by Melkonyan and Pingle (2010). Using this approach, what is the value of each venue? [HINT: Each value depends on the degree of ambiguity λ as well as the parameter from part (c).]

3) [35 points] Consider a religious group with n members. Suppose that member 1 receives utility

$$U_1 = \sqrt{y_1} + (1/n)(\sqrt{r_1} + \sqrt{r_2} + \ldots + \sqrt{r_n})$$

where y_1 is the income of member 1, and r_j is the religious participation of member j. Further assume that member 1's income y_1 equals w_1h_1, where w_1 is wage for member 1, and h_1 is the number of hours worked by member 1. Finally, assume that member 1 faces the time constraint $r_1 + h_1 = T$.

a) Holding constant the religious participation of every other member (r_j for j = 2, …, n), derive the optimal religious participation (r_1*) for member 1. Discuss how r_1* depends on the number of members in the group (n), member 1's own wage (w_1), and the total time available (T).

b) Assuming that the other members (j = 2, …, n) face similar optimization problems, you can use the answer to part (a) to obtain the optimal religious participation (r_j*) for every other member. (Assume that each member j has wage w_j and faces the time constraint $r_j + h_j = T$.) Further assuming that the group has 3 members (i.e., n = 3), that all members have wage equal to 1 (i.e., $w_1 = w_2 = w_3 = 1$), and that T = 40, compute the (numerical) utility level for each member (assuming all are making optimal choices).

c) Is the outcome in part (b) Pareto optimal? If not, would it be better for members increase or decrease their religious participation? Briefly explain.

d) Suppose that the religious group could forbid its members from working, essentially causing all member's wages to fall to zero (i.e., $w_1 = w_2 = w_3 = 0$). Compute the new (numerical) utility level for each member. Are utilities higher or lower than in part (b)? Briefly discuss.

4) [15 points] Briefly summarize the answers given by Iannaccone, Finke, and Stark (*Economic Inquiry* 1997) to each of the following questions.

a) Why did religious participation rise in Japan after World War II?
b) Why did televangelists become more popular in the US after 1960?
c) Why did Asian religions become more popular in the US during the 1960s and 70s?

Econ 451 Exam 3 Spring 2014 Solutions

1a) [10 pts] The worker chooses e to maximize

$$U = 10\ w\ e - e^2$$

Differentiating the utility function with respect to e,

$$dU/de = 10\ w - 2\ e = 0$$

and thus e* = 5 w.

b) [10 pts] A slacker chooses e to maximize

$$U(S) = (0.4)\ [10\ w\ e - e^2\ ] + (0.6)\ [5 - (e - 5)^2]$$

Differentiating with respect to e,

$$dU(S)/de = (0.4)\ [10\ w - 2\ e] + (0.6)\ [-2(e - 5)] = 0$$

and thus e*(S) = 2w + 3

A hard worker chooses e to maximize

$$U(H) = (0.4)\ [10\ w\ e - e^2] + (0.6)\ [-(e - 15)^2]$$

Differentiating with respect to e,

$$dU(H) = (0.4)\ [10\ w - 2\ e] + (0.6)\ [-2(e - 15)] = 0$$

and thus e*(H) = 2w + 9

c) [10 pts] Given wage w = 2, the worker chooses e*(S) = 7 if she becomes a slacker, and chooses e*(H) = 13 if she becomes a hard worker. This implies utility levels

$$V(S) = (0.4)\ [10 * 2 * 7 - 7^2\ ] + (0.6)\ [5 - (7 - 5)^2] = 37$$
$$V(H) = (0.4)\ [10 * 2 * 13 - 13^2] + (0.6)\ [-(13 - 15)^2] = 34$$

and thus the worker chooses to become a slacker and sets e = 7.

1d) [10 pts] Given wage w = 3, the worker chooses e*(S) = 9 if she becomes a slacker, and chooses e*(H) = 15 if she becomes a hard worker. This implies utility levels

$$V(S) = (0.4)\,[10 * 3 * 9 - 9^2\,] + (0.6)\,[5 - (9 - 5)^2] = 69$$
$$V(H) = (0.4)\,[10 * 3 * 15 - 15^2] + (0.6)\,[-(15 - 15)^2] = 90$$

and thus the worker chooses to become a hard worker and set e = 15.

e) [10 pts] The change in I(H) has no effect on the optimal effort choices e*(S) and e*(H), and no effect on the indirect utility level V(S). Given w = 2,

$$V(H) = (0.4)\,[10 * 2 * 13 - 13^2] + (0.6)\,[3 - (13 - 15)^2] = 35.8$$

which is less than V(S) = 37. Thus, the worker remains a slacker with effort e = 7. Given w = 3,

$$V(H) = (0.4)\,[10 * 3 * 15 - 15^2] + (0.6)\,[3 - (15 - 15)^2] = 91.8$$

which is greater than V(S) = 69. Thus, the worker remains a hard worker with effort e = 15. Overall the program had no effect: it didn't increase V(H) enough to turn slackers into hard workers, and didn't influence the effort choice of workers who were already hard workers.

2a) [6 pts]

$$E\pi(S) = (1/5)\,15 + (2/3)\,15 + (2/15)\,15 = 15$$
$$E\pi(M) = (1/5)(-15) + (2/3)\,60 + (2/15)\,60 = 45 \quad \text{(best choice)}$$
$$E\pi(H) = (1/5)(-175) + (2/3)(-100) + (2/15)\,200 = -75$$

b) [10 pts]

venue	best possible outcome	worst possible outcome
S	15	15 (max-min choice)
M	60	–15
H	200 (max-max choice)	–175

c) [12 pts] The Hurwicz criterion is a weighted average of the max-min and max-max values, where the weight α reflects the pessimism of the decision-maker.

$$V(S) = \alpha\,15 + (1-\alpha)\,15 = 15$$
$$V(M) = \alpha\,(-15) + (1-\alpha)\,60 = 60 - 75\alpha$$
$$V(H) = \alpha\,(-175) + (1-\alpha)\,200 = 200 - 375\alpha$$

S is the best option when pessimism is high ($\alpha > 3/5$), M is the best option when pessimism is moderate ($7/15 < \alpha < 3/5$), and H is the best option when pessimism is low ($\alpha < 7/15$).

2d) [10 pts] Melkonyan and Pingle assume that partial ambiguity implies a set Q of probability distributions over outcomes. At one extreme, when the degree of ambiguity (λ) falls to zero, the set Q collapses to a unique anchor distribution. At the other extreme, when ambiguity is complete ($\lambda = 1$), the set Q expands to include every possible probability distribution. Formally, the value of each option i is equal to

$$V(i) = \alpha \min_{q \in Q} \{ \sum_j q_j u_{ij} \} + (1-\alpha) \max_{q \in Q} \{ \sum_j q_j u_{ij} \}$$

One can show that this is equivalent to

$$V(i) = (1-\lambda) \sum f_j u_{ij} + \lambda \{ \alpha \min_j u_{ij} + (1-\alpha) \max_j u_{ij} \}$$

where λ is the degree of ambiguity, α is the degree of optimism, and $f = (f_1, \ldots, f_n)$ is the anchor distribution. Thus, the value function is a weighted average of the expected value given the anchor distribution (first term) and the Hurwicz criterion (second term).

For the current problem, if we assume the distribution from part (a) is the anchor distribution,

$$V(S) = (1-\lambda)\, 15 + \lambda\, (\alpha\, 15 + (1-\alpha)\, 15\,)$$
$$V(M) = (1-\lambda)\, 45 + \lambda\, (\alpha\, (-15) + (1-\alpha)\, 60)$$
$$V(H) = (1-\lambda)\, (-75) + \lambda\, (\alpha\, (-175) + (1-\alpha)\, 200\,)$$

3a) [13 pts] Substituting member 1's constraints into her utility function,

$$U_1 = \sqrt{w_1 (T - r_1)} + (1/n) \sum_{j=1}^{n} \sqrt{r_j} \quad .$$

Differentiating with respect to r_1 and setting this derivative equal to zero,

$$\frac{dU_1}{dr_1} = \frac{-\sqrt{w_1}}{2\sqrt{T - r_1}} + \frac{1}{2n\sqrt{r_1}} = 0 \quad .$$

Rewriting this equation, we obtain the optimal solution

$$r_1 * = \frac{T}{1 + w_1 n^2} \quad .$$

Intuitively, r_1* falls as n rises (because the free-rider problem worsens), r_1* falls as w_1 rises (because the individual has better non-religious opportunities), and r_1* rises as T rises (because the individual has more time available).

3b) [8 pts] If other members face similar problems, each member j's optimal participation is given by

$$r_j * = \frac{T}{1 + w_j n^2} \quad .$$

Assuming n = 3, T = 40, and w_j = 1 for all j, we thus obtain r_j* = 4 for each member j. Substituting these r_j*'s into any member's utility function,

$$U_j = \sqrt{(1)(40-4)} + (1/3)(3)\sqrt{4} = 6 + 2 = 8 \quad .$$

c) [6 pts] No, the outcome is not Pareto optimal. Religious participation generates positive externalities and thus members set their participation too low. To illustrate, suppose that every member increased r slightly from 4 to 5. Each member's utility would increase from 8 to $\sqrt{35} + \sqrt{5} \approx 8.15$.

d) [8 pts] Given w = 0, each member would spend all of her time in religious participation, and each member's utility would fall to $\sqrt{40} \approx 6.32$. As we saw in lecture, given the positive participation externality, it is sometimes possible for religious groups to increase members' utility by decreasing their non-religious opportunities. But in this example, given initial wage w = 1, decreasing the wage to w = 0 would hurt members. However, if w was initially very close to zero (less than 1/9), you can show that decreasing w would actually raise members' utilities.

4a) [5 pts] The state religion (Shinto) lost its monopoly status, allowing other religions to compete for members.

b) [5 pts] Prior to 1960, the FCC required television networks to carry free religious programming, which was controlled by (low strictness) mainline Protestant denominations. After 1960, this regulation was abolished, allowing televangelists (from more strict denominations) to compete for viewers.

c) [5 pts] While other researchers have suggested changes on the demand side of the religious market (the "consciousness revolution"), Iannaccone et al focus on the supply side. They argue that relaxation of immigration laws allowed more Asian teachers to move to America and compete for followers.